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Title: Classes of operators in semi-Hilbertian spaces
Abstract: We consider a Hilbert space H with an additional semi inner product defined by a positive semidefinite operator A, namely $< \xi, \eta >_A = < A\xi, \eta >$ for every $\xi, \eta \in H$. Our goal is to describe classes of operators which are Hermitian, isometric, unitary or partially isometric with respect to $< \ . \ , \ . >_A$. It must be observed that this additional structure induces an adjoint operation. However, this operation is defined for not every bounded linear operator on H, unless A is invertible. Operators T which are selfadjoint with respect to $< \ . \ , \ . \ >_A$ in the sense that $< T\xi, \eta >_A = < \xi, T\eta >_A$ for every $\xi, \eta \in H$ are called "symmetrizable" (with respect to A) and they have been studied since longtime. For those operators T which admit an adjoint with respect to $< \ . \ , \ . \ >_A$, we choose one, denoted by $T * A$, which has similar, but not identical, properties as the classical T^*.

Since not every operator admits an A- adjoint and, in case it admits one, it may have many others, then the extensions of isometries, unitary and partial isometries are not trivial. Many of the descriptions are easier if the range $R(A)$ is closed. Moreover, for describing the class of A-partial isometries which is our main goal, we need an additional hypothesis, known as compatibility in the recent literature. A closed subspace K and a positive (semidefinite) operator A are called compatible if there exists a (bounded linear) projection Q onto K which is A- selfadjoint.

We shall describe elsewhere the relationship between the system generated by H, A and the adjoint operation $^\sharp$ as studied here, with the Hilbert space $R(A^{1/2})$ with the inner product $< A^{1/2}\xi, A^{1/2}\eta >' = < P\xi, P\eta >$, where $\xi, \eta \in H$ and $P = P_{\overline{R(A)}}$. These Hilbert spaces are relevant in the De Branges theory.