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Title: An analogue of the Riesz-Haviland Theorem for the truncated moment problem

Abstract. Let $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{|i| \leq 2n}$ denote a d -dimensional real multisequence, let K denote a closed subset of \mathbb{R}^d , and let $\mathcal{P}_{2n} := \{p \in \mathbb{R}[x_1, \dots, x_d] : \deg p \leq 2n\}$. Corresponding to β , the *Riesz functional* $L \equiv L_\beta : \mathcal{P}_{2n} \rightarrow \mathbb{R}$ is defined by $L(\sum a_i x^i) := \sum a_i \beta_i$. We say that L is *K-positive* if whenever $p \in \mathcal{P}_{2n}$ and $p|_K \geq 0$, then $L(p) \geq 0$. In joint work with Lawrence A. Fialkow, we prove that β admits a K -representing measure if and only if L_β admits a K -positive linear extension $\tilde{L} : \mathcal{P}_{2n+2} \rightarrow \mathbb{R}$. This provides a generalization (from the full moment problem to the truncated moment problem) of the Riesz-Haviland Theorem. We also show that a semialgebraic set solves the truncated moment problem in terms of natural “degree-bounded” positivity conditions if and only if each polynomial strictly positive on that set admits a degree-bounded weighted sum-of-squares representation.