María Celeste Gonzalez, Instituto Argentino de Matemática.
Title: Metric properties of projections in semi-Hilbertian spaces
Abstract: Let H be a Hilbert space, $L(H)$ the algebra of bounded linear operators on H and $\mathcal{Q}$ the subset of $L(H)$ of all projections (i.e. idempotents). Given a closed subspace K of H, $\mathcal{Q}\ K$ denotes the subset of $\mathcal{Q}$ of all projections with image K. Let $P_K \in \mathcal{Q}_K$ denote the unique Hermitian projection with image K. The following properties are well known:

(I) For all $0 \neq Q \in \mathcal{Q}$ it holds $\|Q\| = 1$ if and only if $Q^* = Q$;

(II) For every non trivial $Q \in \mathcal{Q}$ it holds $\|Q\| = \|I - Q\|$;

(III) Given closed subspaces K and L of H it holds $\|P_K - P_L\| \leq \|Q_K - Q_L\|$ for every $Q_K \in \mathcal{Q}_K$ and $Q_L \in \mathcal{Q}L$;

(IV) For all closed subspaces K and L of H it holds $\|P_K - P_L\| \leq 1$. Equality holds if and only if P_K and P_L commute;

(V) For all closed subspaces K and L of H it holds $\|P_K - P_L\| = \max\{\ \|P_K(I - P_L)\|,\ \|P_L(I - P_K)\|\ \}$;

(VI) For every $Q \in \mathcal{Q}$ it holds $\|Q\| = \dfrac{1}{\sin\theta}$ if $\theta \in [0, \pi/2]$ is the angle such that $\cos\theta = \sup\{| < \xi, \eta > | : \xi \in R(Q), \eta \in N(Q) \text{ and } \|\xi\| = \|\eta\| = 1\}$.

The main goal of this talk is to study these properties if we consider an additional seminorm $\|\ .\ \|_A$, defined by a positive semidefinite operator $A \in L(H)$ by $\|\xi\|_A^2 =< A\xi, \xi >$, $\xi \in H$, and we replace the operator norm in formulas (I) to (VI) by the quantity

$$\|T\|_A = \sup\{\|T\xi\|_A : \ \|\xi\|_A = 1\}.$$