U. Haagerup, Odense University.
Title of Taft Center Lecture: The invariant subspace problem -Linear algebra in infinitely many dimensions
Abstract: Most scientists have at some point in their life been introduced to linear algebra in finitely many dimensions, i.e. calculation with n x n matrices, including determinants, eigenvalues and eigenvectors of such matrices. The development of quantum physics in the 20's have forced us to work with linear maps (= operators) on infinite dimensional vector spaces in order to get a satisfactory description of the world around us. This development formed the basis of the mathematical field "Operator Algebra Theory", which was initiated by von Neumann and his student Murray in the late 30's.

A famous and still open problem in this theory is the invariant subspace problem: "Does every bounded operator on a Hilbert space have a non-trivial closed invariant subspace ?" In this lecture I will discuss this problem as well as a variant of the problem coming from a special class of von Neumann algebras, the so-called factors of Type II_1. Although these algebras are infinite dimensional, they share many properties with the algebra of n x n matrices: For instance all operators in such an algebra has a generalized determinant (Fuglede and Kadison 1952) and a generalized eigenvalue distribution (L. G. Brown 1983). In a recent joint work with Hanne Scultz, we give a partial solution to the invariant subspace problem for II_1 factors by providing a new construction of spectral subspaces (generalized eigenspaces) for all operators in such a factor.